From Hades to Helios:
Bringing the Hour Angle Solar Back into the Light

Hakeeb A. Nandalal, T.T.L.S.

September 2015

Abstract

Trinidad & Tobago Land Surveyors almost exclusively use the Altitude Method for Solar Azimuth because it does not require accurate time-keeping. With the availability of accurate time information from GPS signals and in-built powerful computational capability, the use of Android devices now make it possible to re-introduce the Hour Angle Method for the determination of Azimuth from Solar Observations for Cadastral Surveys.

1. Comparing the Altitude and Hour Angle Methods

Rather than re-invent the wheel, the author borrows liberally from "Revenge of the Altitude Solar" by Jerry L. Wahl:

Altitude Method

The altitude method requires:

Latitude of the observation point.
Rough Longitude of the observation point.
Time of pointing (nearest minute may suffice).
Horizontal Angle (and simultaneous) Vertical Angle to Sun at moment of pointing.

Local atmospheric conditions of absolute pressure, (elevation may be used as a second choice), and air temperature.

The following values are then computed:

The Sun's Declination: determined from the date/time using an ephemeris (not highly critical for time).
Refraction is a function of the observed pressure and temperature.
Azimuth of the Sun: computed with the Altitude formulae.

The advantages of the altitude method include a minimal need for accurate time or determination of longitude. The disadvantages are that it requires a more complex pointing procedure to obtain both the horizontal and zenith readings to the Sun, and it requires some determination of atmospheric conditions in order to figure the refraction correction. In addition the refraction determination is to some degree an educated guess.

Hour Angle Method

The hour angle method requires:

Latitude of the observation point.
Very Accurate Longitude of the observation point.
Very Accurate (sub second) time of pointing.
Horizontal Angle (only) to Sun at moment of pointing.

The following are then computed:

The Sun's Declination: determined from the date/time using an ephemeris.
Local Hour Angle computed from:
Time
Longitude
Greenwich Hour Angle (GHA) of Sun from an ephemeris (highly critical for time).
Azimuth of the Sun: computed with the Hour Angle formulae.

The advantages of the hour angle method include the fact that there is no need to observe the Vertical Angle to the Sun. This results in simpler pointings as compared to the altitude method. It also allows a successful observation under some conditions where the solution to an altitude observation is highly unstable. The disadvantage is that it requires extraordinary care in the time determination, such as: using time signals, having an accurate watch, applying proper correction to UT1, developing good technique in reaction time with watch.

2. Addressing Traditional Limitations with Android Technology

Location

Both methods require the determination of Latitude & Longitude. Up to the late 1990s this was usually done with the aid of a printed map/ward sheet. Then came the hand-held GPS receiver accurate to ±0.1” geodetic, equivalent to ±5m on ground under good conditions. The majority of Android devices now have GPS sensors capable of the equal accuracies.

Timing

This is the singular aspect of the Hour Angle method that made it unpalatable to surveyors. Again pre-Android, getting accurate time meant the use of shortwave radio time signals and chronometers which were not practical for everyday field observations. Now it is possible to determine the correction to the device clock via not just one but two separate and independent methods.

The first method is through the use of Internet Time Servers like time.nist.gov. One way to access this service is an App called “*UTC Time*” available online. The App gives you the time on your device, the time from the server and time correction. Web sites such as *http://time.is* also give the time correction. The delay in accessing the time server is also stated on the web page and it is usually under 0.1s. Note that internet access is required to ‘ping’ time servers so for remote locations away from Wi-Fi, cellular service *is* required.

The second method is from the GPS sensor in Android devices. Location is determined from the differences between the times the signals are sent from the GPS satellites and the when they are received. A Global Positioning System (GPS) receiver measures its distance to GPS satellites based on how long it takes for a radio signal to arrive from each satellite and from these distances calculates the receiver’s 3D position. Because radio waves travel at 299,792,458 m/s (the speed of light, c) these measurements of nanoseconds must be very precise. Each GPS satellite contains multiple atomic clocks that give very precise time data to the GPS signals. Therefore not only is the accurate location of the receiver known but we get accurate time as well. Note when using GPS time, cellular service *is not* required.

Ephemeris

From *http://en.wikipedia.org/wiki/Ephemeris*

In astronomy and celestial navigation, an ephemeris (plural: ephemerides; from Latin ephemeris, "diary" of Greek origin meaning "diary, calendar") gives the positions of naturally occurring astronomical objects in the sky at a given time or times. Historically, positions were given as printed tables of values, given at regular intervals of date and time. Modern ephemerides are often computed electronically from mathematical models of the motion of astronomical objects and the Earth. Even though the calculation of these tables was one of the first applications of mechanical computers, printed ephemerides are still produced, as they are useful when computational devices are not available.

Surveyors consider “computational devices” to be generally either of two extremes: the traditional HP calculator or full-blown Windows PCs. The former being incapable of computing complex values like “nutation in obliquity” (500+ lines of Java code) and the latter, although now on Surface Tablets, are not yet practical for field work. The computational device being ignored is the one running the Android operating system. The CPUs in these devices are more than capable for computing values traditionally obtained from printed ephemeris like Declination (δ) and Greenwich Hour Angle (GHA) both of which can be computed to sub-second accuracy from two inputs: date & time.

3. Altitude Method: How Accurate is ‘to the Nearest Minute’?

In section 1, Jerry Wahl stated that for the Altitude Method, the “nearest minute may suffice”. In this case the time to the nearest minute is required for computing the declination of the sun. But we need to put this to the test. What happens if our time is off by (say) 5 minutes? What effect does this ultimately have on the final Azimuth? One way to go about this is through calculus but another practical way is simple to compute an Azimuth and then change the time by 5 minutes and re-compute. With the aid of the webpage *http://www.survtools.com/solar/solar.htm* written by the author, this exercise took just a few minutes. The date of the demo problem was set to the 15th of each month of 2015 and the time was set to 08:00 and 08:05 local time in order to simulate a 5-minute timing error. The Table in Fig.1 gives the changes in seconds of arc. Fig.2 is a graph of the data from Fig.1.

Month	Declination at 08.00	Declination at 08.05	Change in Dec. “	Change in Azimuth “
Jan	-21° 07' 35.8"	-21° 07' 33.5"	2.3	-03
Feb	-12° 41' 21.6"	-12° 41' 17.4"	4.2	-05
Mar	-2° 09' 14.2"	-2° 09' 09.3"	4.9	-05
Apr	+9° 45' 00.9"	+9° 45' 05.4"	4.5	-04
May	+18° 51' 16.1"	+18° 51' 19.0"	2.9	-04
Jun	+23° 18' 08.6"	+23° 18' 09.1"	0.5	-01
Jul	+21° 31' 26.3"	+21° 31' 24.3"	-2.0	03
Aug	+14° 03' 26.8"	+14° 03' 22.9"	-3.9	05
Sep	+3° 02' 25.2"	+3° 02' 20.4"	-4.8	06
Oct	-8° 29' 53.9"	-8° 29' 58.5"	-4.6	05
Nov	-18° 28' 01.2"	-18° 28' 04.4"	-3.2	04
Dec	-23° 15' 31.7"	-23° 15' 32.4"	-0.7	01

Fig.1 Effect on Declination & Azimuth by 5-Minute Change in Time

We can infer from Figs.1&2 that a 5-minute timing error would result in less than a 10" error in the final Azimuth using the Altitude Method. This of course is acceptable for cadastral surveys in Trinidad & Tobago where bearings are stated to the nearest minute of arc on cadastral plans.

It is interesting to note from the graph in Fig.2 that the greatest changes to Azimuth occur during the vernal/autumnal equinoxes (March/September) and the smallest changes occur during the summer/winter solstices (June/December) in the Northern Hemisphere.

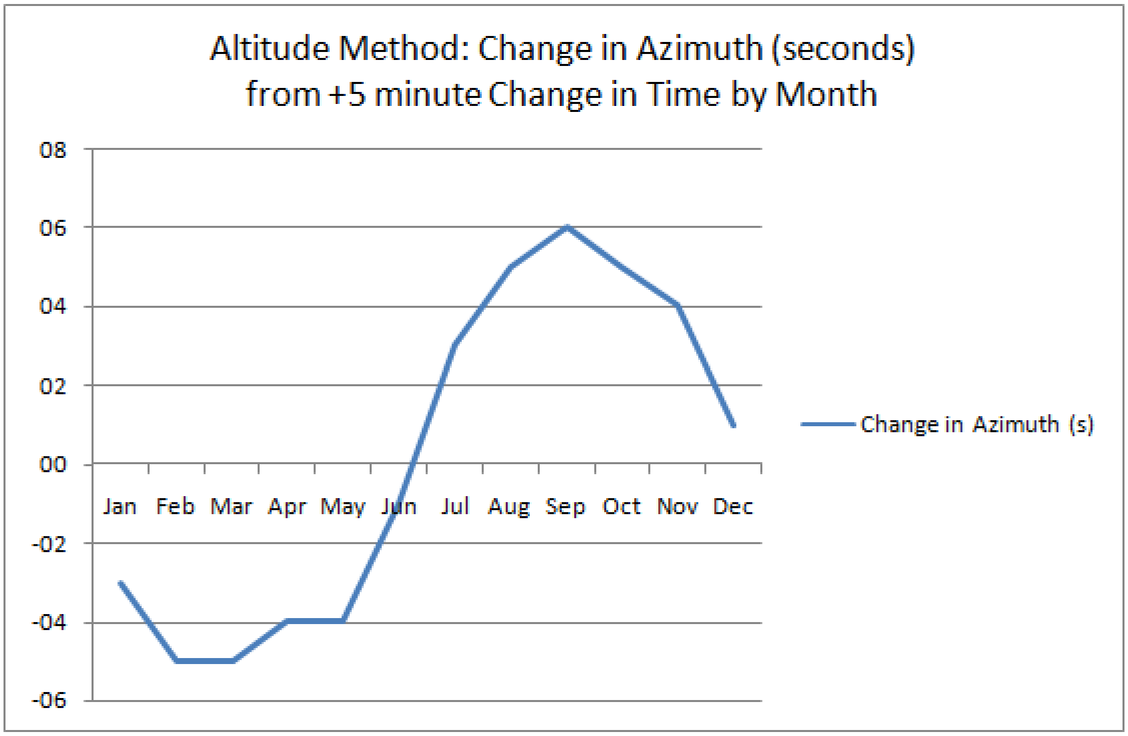

Fig.2 Altitude Method: Effect on Azimuth by 5-Minute Change in Time

4. Hour Angle Method: Is Sub-Second Timing Really Necessary for Cadastral Work?

In section 1, Jerry Wahl also stated that for the Hour Angle Method, the "Accurate (sub second) time of pointing" is a requirement. This may be true for high-order control surveys but is it really necessary for cadastral surveys in Trinidad & Tobago? The author tested this question using the webpage, written by the author, at *http://www.survtools.com/solar-ha/solar-ha.htm*. This time the date of the demo problem was set to the 15th of each month of 2015 and the time was set to 08:00:00 and 08:00:05 local time in order to simulate a 5-second timing error.

The Table in Fig.3 gives the changes to the final Azimuth in seconds of arc. Fig.4 is a graph of the data from Fig.3.

Month	Change in Azimuth (arc seconds)
Jan	21
Feb	19
Mar	16
Apr	11
May	5
Jun	3
Jul	4
Aug	8
Sep	15
Oct	21
Nov	25
Dec	23

Fig.3 Hour Angle Method: Effect on Azimuth by a 5-Second Change in Time

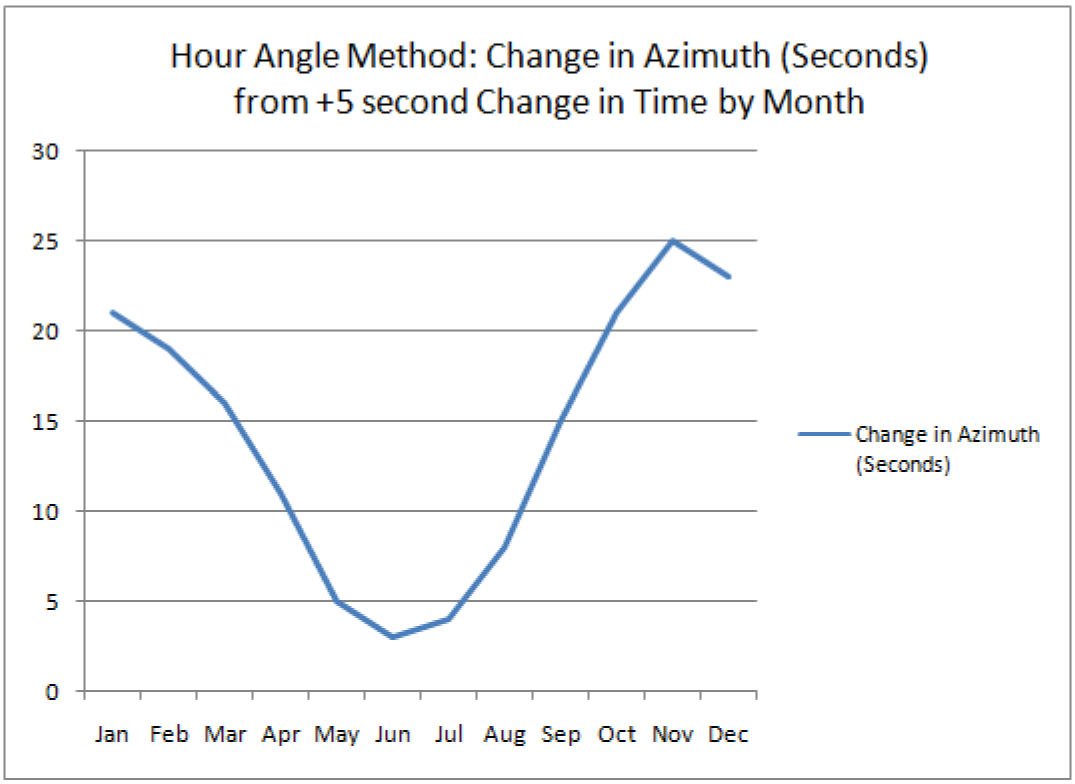

Fig.4 Hour Angle Method: Effect on Azimuth by a 5-Second Change in Time

We can infer from Figs.3&4 that a 5-second timing error would result in less than a 30” error in the final Azimuth using the Hour Angle Method. Although acceptable for cadastral surveys, most Trinidad & Tobago Land Surveyors would probably prefer to keep the error under 10” of arc which is easily achieved by keeping the timing error to less than 2 seconds between September-March and to less than 5 seconds between April-August. However at no time of the year is sub-second accuracy of time required for cadastral work.

It is interesting to note from the graph in Fig.4 that the greatest changes to Azimuth occur near the winter solstice (December) and the smallest changes occur near the summer solstice (June) in the Northern Hemisphere.

5. Hour Angle Method: The Spherical Geometry

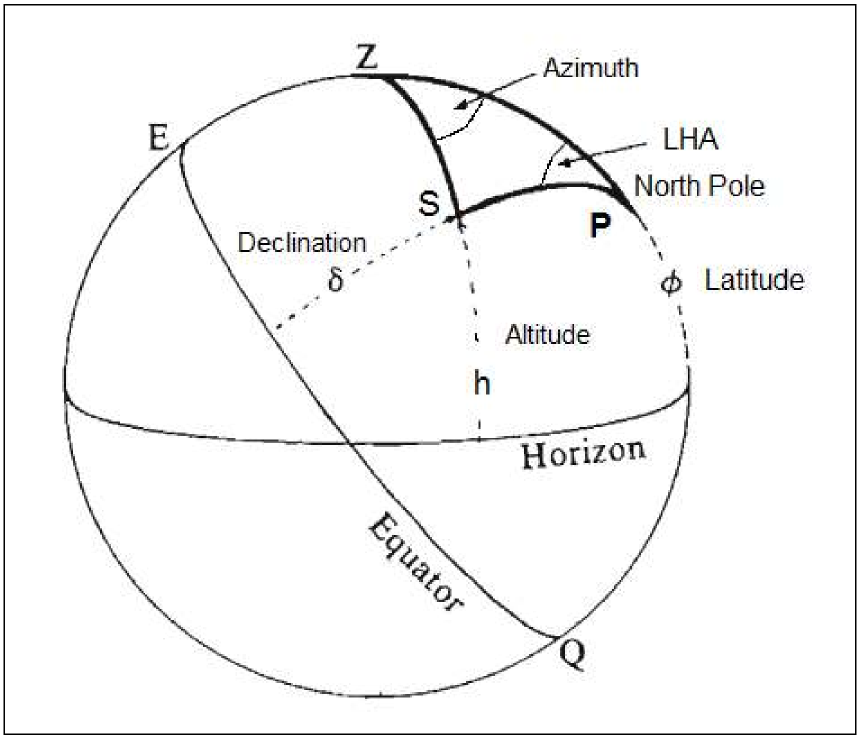

Fig.5 The Astronomical Triangle

Since a spherical triangle is comprised of six angles (three sides & three internal angles), to solve for any unknown requires just three known values. For the Altitude Method the three knowns are co-declination (SP in Fig.5), co-latitude (ZP) and zenith angle (ZS) from which we solve for Azimuth by:

$$A = \cos^{-1} [(\sin \delta - \sin \phi \sin h) / (\cos \phi \cos h)]$$

Note: 360° - A for PM obs.

Observed h corrected for: Atmospheric Refraction − 58” cot h and Horizontal Parallax + 8.8” cos h

For the Hour Angle Method the three knowns are co-declination (SP in Fig.5), co-latitude (ZP) and Local Hour Angle (ZPS) from which we solve for Azimuth by:

$$A = \tan^{-1} [(\sin LHA / (\cos LHA \sin \phi - \tan \delta \cos \phi)]$$

Where 0° < LHA < 180° & A is +ve : Add 180° to A
Where 0° < LHA < 180° & A is -ve : Add 360° to A
Where 180° < LHA < 360° & A is -ve : Add 180° to A

Whereas there is a single measureable time correction in the Hour Angle Method, there are three possible errors in vertical angle measurement for the Altitude Method: instrumental (measurable with calibration), atmospheric refraction (estimated) and horizontal parallax (estimated).

6. Computing Local Hour Angle

Local Hour Angle is computed from the Greenwich Hour Angle (previously obtained from printed ephemeris, now computed from time directly on Android) and the observer's longitude using the following formula for longitudes West of Greenwich:

$$LHA = GHA_{Sun} - Longitude_{w}$$

where $Longitude_{w}$ is a positive western value.

For example if the GHA_{Sun} is 134° 41' 28" and the observer's longitude is 61° 28' 22" W then the LHA = 134° 41' 28" - 61° 28' 22" = 73° 13' 06".

7. Computing the Sun's Semi-Diameter Correction

Since observations to the sun are made to both the left edge (left limb) and the right edge (right limb), we need to compute the semi-diameter correction to find the Azimuth to the sun's vertical centreline. For this we need the altitude of the sun (h) without observing it directly, therefore we are going to compute it from the known values of the astronomical triangle as follows:

$$h = \sin^{-1} [\sin \phi \sin \delta + \cos \phi \cos \delta \cos LHA]$$

We also need the sun's semi diameter (SD), which again is normally found in the printed ephemeris, but can be computed with the appropriate formulae. On the horizon SD ≈ 16'.

$$\text{Semi-Diameter Correction} = SD / \cos h$$

The sun semi-diameter correction must be added to the left-limb horizontal reading and subtracted from the right-limb horizontal reading to correct these readings to the sun's vertical centreline. It should be noted that the semi-diameter correction increases with time in the morning and decreases with time in the afternoon as the apparent size of the sun increases and decreases respectively. Very rarely is h exactly 90° so check for division by zero.

8. One More Time: Terrestrial Time

The time used to compute ephemeris data such as GHA_{Aires} and RA_{Sun} ($GHA_{Sun} = GHA_{Aires} - RA_{Sun}$) is Terrestrial Time (TT). However the time we get from clocks is Universal Time (UT) which when corrected for polar wander gives UT1. The terms UT and UT1 are generally considered interchangeable. A final value called Delta T (ΔT) when added to UT1 gives TT. ΔT was defined to be 32.184 seconds at 0h UT on January 1st 1977 and 65.7768 seconds at 0h UT on January 1st 2009. There are several formulae developed to predict ΔT in seconds of time. The author chose those developed by Espenak & Meeus for software implementation:

$$u = (Year - 2000.0) / 100.0$$
$$\Delta T = 62.92 + u * (32.217 + 55.89 * u)$$

These equations are valid for the period 2006 to 2050 according to Espenak & Meeus.
Note a "leap second" was added to UT on 30th June 2015 at 11:59.59 p.m. June 30th was a second longer because the rotation of the earth is slowing down.

9. From True North to Grid North

The Azimuth of the Sun computed from the Astronomical Triangle is relative to Celestial North Pole and is therefore relative to True North. However cadastral plans in Trinidad & Tobago are plotted using either of two map projections, the Universal Transverse Mercator (UTM) or the Cassini-Soldner. The Central Meridian (λ_o) for the UTM projection is 63° W (Trinidad & Tobago) whereas λ_o for the Cassini projection is 61° 20' W (Trinidad) and 60° 41' 09.632" W (Tobago). The difference between the Grid and True Azimuth is the Grid Convergence Angle (C in Fig.6). C is added to the True Azimuth to get the Grid Azimuth (also called the Grid Bearing).

The Grid Convergence Angle and Grid Azimuth are computed by:

$$C = (\lambda - \lambda_o) \sin\phi = \Delta\lambda \sin\phi$$

$$\text{Grid Azimuth} = \text{True Azimuth} + C$$

C for the Cassini projection is always greater numerically than C for the UTM projection therefore Cassini Azimuths are always greater than UTM Azimuths for the same line. The difference between Cassini and UTM Grid Azimuth at a point with latitude ϕ can be computed by:

$$\text{Grid Azimuth Difference} = (\lambda_{o\,UTM} - \lambda_{o\,Cassini}) \sin\phi$$

Note the point's longitude λ is *not* part of the calculation.

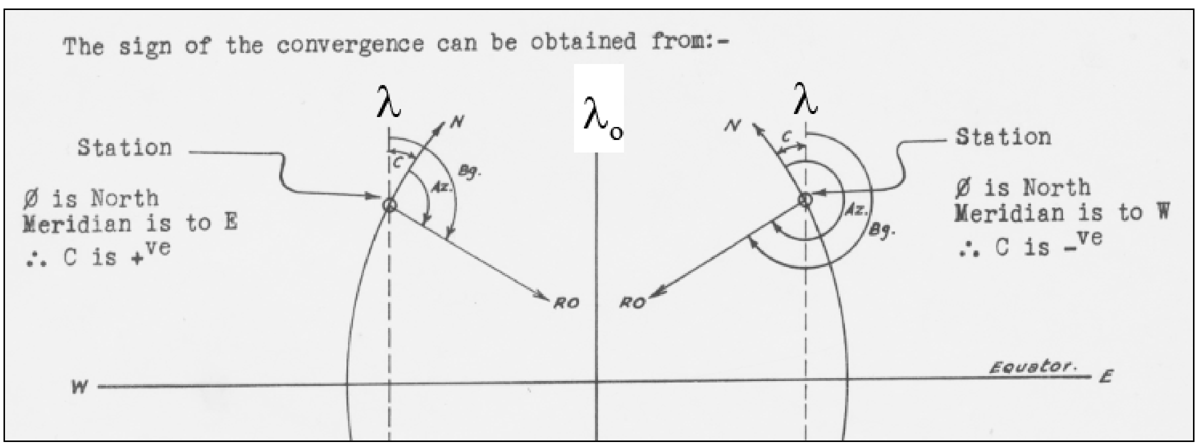

Fig.6 Grid Convergence Angle on both Sides of the Central Meridian

Worked Example: Location Chacachacare Island

ϕ = 10° 41' 31" N ; λ = 61° 45' 10" W ; True Azimuth R.O. = 297° 20' 21"

Cassini: $C = (\lambda - \lambda_o) \sin\phi$ = (61° 45' 10" – 61° 20') sin[10° 41' 31"] = +0° 04' 41"
Cassini Azimuth R.O. = 297° 20' 21" + 0° 04' 41" = 297° 25' 02"

UTM: $C = (\lambda - \lambda_o) \sin\phi$ = (61° 45' 10" – 63°) sin[10° 41' 31"] = –0° 13' 52"
UTM Azimuth R.O. = 297° 20' 21" – 0° 13' 52" = 297° 06' 29"

Cassini Azimuth – UTM Azimuth = $(\lambda_{o\,UTM} - \lambda_{o\,Cassini}) \sin\phi$ = (63° – 61° 20') sin[10° 41' 31"] = 18' 33"
Check#1: Azimuth Difference 297° 25' 02" – 297° 06' 29" = 18' 33"
Check#2: Grid Convergence Angle Difference 0° 04' 41" – (–0° 13' 52") = 18' 33"

10. Hour Angle Method: Observations and Input Data

The following observations and input data are required to compute Azimuth using the Hour Angle method:

Observations	Input Data
Horizontal Reading to R.O.	Latitude on Naparima Datum
Horizontal Reading to Sun Left/Right Limbs	Longitude on Naparima Datum
Local Time of Sun Observations	U.T. Offset & Time Correction

Based on the lists above it is clear to say that for determining Azimuth by solar observations for cadastral surveys, all a Trinidad & Tobago Land Surveyor would require is a total station (with a solar filter) and a GPS-equipped Android device with the appropriate software such as the "*Solar HA*" app written by the author which can be downloaded at *http://www.survplan.com/android*.

Solar Azimuth by Hour Angle: Example from Field Exercise

```
DAY:   08                         LATITUDE:     10° 29' 42.5" N     STATION: N1
MONTH: 02                         LONGITUDE:    61° 22' 52.9" W     R.O.:    T1
YEAR:  2015

R.O. HORIZ READING:               000° 00' 00"
TIME CORR.:                       +00h 00m 00.9s

LOCAL TIME:                       07h 19m 24.6s
SUN LIMB:                         LEFT
SUN HORIZ READING:                99° 51' 23"
--------------------------------------------------------------
CORRECTED LOCAL TIME:             07h 19m 25.5s
CORRECTED U.T.:                   11h 19m 25.5s
DeltaT:                                   69.0s
GHA SUN:                          346° 19' 29"
LHA SUN:                          284° 56' 36"
DECLINATION SUN:                  -15° 00' 41"
SUN SEMI-DIAMETER CORR.:          000° 16' 32"
SUN CL HORIZONTAL:                100° 07' 55"
SUN CL TO RO ANGLE:               259° 52' 05"
AZIMUTH SUN CL (True):            107° 49' 25"
AZIMUTH RO (True):                007° 41' 30"
CASSINI GRID CONVERGENCE:         +00° 00' 31"
UTM GRID CONVERGENCE:             -00° 17' 41"
R.O. Cassini Azimuth:               7° 42' 01"
R.O. U.T.M. Azimuth:                7° 23' 48"
--------------------------------------------------------------
UTM Results from 4 "Sets" of Readings of Sun Left/Right Limbs
Instrument:  Leica TCR307 5" Total Station (Face Left Only)
Android:     Samsung Galaxy S3 used for Timing and Location

7° 23' 48"   Left
7° 23' 28"   Right
7° 23' 37"   Left
7° 23' 23"   Right
----------
7° 23' 34"   Mean R.O. U.T.M. Azimuth
----------
```

11. A Final Word on Datum Conversions

Throughout this paper the datum used for geodetic position is understood to be the Naparima Datum which is based on the International 1924 ellipsoid. However readers should be aware that the default datum for GPS is WGS84 and the raw position values obtained from Android devices must be converted to the Naparima Datum. The author uses the Standard Molodensky Transformation to achieve this. The parameters for the two ellipsoids are:

Ellipsoid	**WGS84**	**International 1924**
Semi-Major Axis	6378137.0	6378388.0
Flattening	1/298.257223563	1/297

The 3 parameters for the transformation going from WGS84 → Naparima are:

$$\Delta x = +0.216 \qquad \Delta y = -372.252 \qquad \Delta z = -172.231$$

Alternatively, going from WGS84 → Naparima, readers can use the following quick conversion which yields a maximum error of ±0.2" in either value:

$$\phi_{NAP} \approx \phi_{WGS84} - 6.3'' \text{ [North]}$$
$$\lambda_{NAP} \approx \lambda_{WGS84} + 5.9'' \text{ [West]}$$

References

Wahl , Jerry L. *Revenge of the Altitude Solar*
http://www.cadastral.com/papersl1.htm

Wikipedia, *Ephemeris*
http://en.wikipedia.org/wiki/Ephemeris

Recommended Websites

Institut de Mécanique Celeste et de Calcul des Ephemerides
http://www.imcce.fr/en/ephemerides/formulaire/form_ephepos.php

Jet Propulsion Laboratory Ephemeris Generation
http://ssd.jpl.nasa.gov/horizons.cgi

Celestial Navigation by Dr. Henning Umland
http://www.celnav.de

Delta T: Terrestrial Time, Universal Time and Algorithms for Historical Periods
http://www.staff.science.uu.nl/~gent0113/deltat/deltat.htm

Time Is
http://time.is

UTC Time App
https://play.google.com/store/apps/details?id=com.bjg222.utctime&hl=en

■